

# The powerful tool of Queuing Theory in the traffic problem of urban road

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**Abstract** — It is practically significant to research the traffic flow of intersection because the capacity of intersection affects the efficiency of highway network directly. This paper analyzes the traffic conditions of an intersection in certain urban by the methods of queuing theory and statistical experiment, sets up a corresponding mathematical model and compares it with the actual values. The result shows that queuing theory is applied in the study of intersection traffic flow in Urban area and it can provide references for the other similar designs.

**Keywords** — Queuing Theory, Queuing Models, System Metrics, Vehicles (veh).

## I. INTRODUCTION

With the development of economy, vehicles maintain a substantial increase in volume of Myanmar, queuing phenomenon is so common in road traffic. Intersection is the main concentrated area of stream of people and vehicles; also, it is one infrastructure construction that connecting the roads to make it play network functions. In daily life, traffic congestion responses to the intersection directly. Traffic flow intersection is one of the traffic problems. It is so clear that road intersection will be the bayonet of traffic capacity and safety. Therefore, it is significant to study the intersection flow to improve the congested traffic and maintain social order. In the early 20th century, queuing theory originated from the Danish engineer Erlangs study of telephone exchange efficiency of communication system in 1931. After the section world war, especially with the rapid development of computer and communication technology, queuing theory got attention and developed fast, also, it became an important branch of operations research and its corresponding disciplines theory and reliability theory were developed. In the mid-1930-s, queuing theory was recognized one important subject when W.Feller recommended birth and death process. In the early 1950s, D.G.Kendall researched queuing theory systematically by the method of Markov chain and made it develop further. In the 1960s, the projects studied complicatedly in queuing theory, it is so difficult to get the exact solution that people began to study the approximate method.

In the traffic engineering, 1936, Adams considered the pedestrian delay problem by queuing theory that the intersection which not set the traffic signals then queuing theory had been widely used in traffic control. Such as the study of vehicle delay, traffic capacity, configuration light time, the design and management of traffic facilities for the park and station and so on. At present, queuing system model has been widely used in all kinds of management system. Such as production management, inventory management, business management, transportation, banking, medical services, computer design and performance evaluation, and so on.

## II. BASIC KNOWLEDGE

Queuing theory is the mathematical theory and method of queuing system (stochastic system). In daily life, people will encounter all sorts of queuing problems, such as, standing at bus stops, going to hospital, and going to the ticket office to buy the tickets and so on. In these problems, bus and passengers, doctor and patients, conductor and the buyers forms a queuing system or service system respectively; the former can be regarded as service agencies and the latter can be regarded as customers. The queue can be tangible queue may also be intangible queue. For example, several passengers make telephone call to order train tickets at the same time, if a passenger is on the phone, can only wait for the other passengers, this form of queue is invisible. The people or some objects can be the queue, such as semi-finished products for processing in the production line, machine waiting for maintenance, and the information waiting for computing center to process, etc. Queuing theory consists of three parts: input process, queuing rules and service agencies. The schematic diagram as follows:

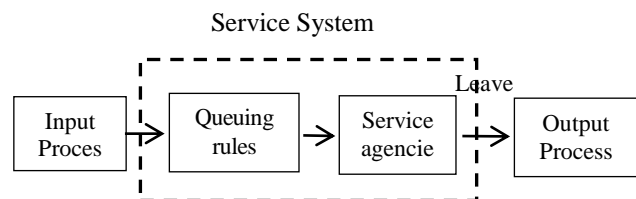


Figure 1. The composition of queuing system

Queuing theory mainly studies three aspects:

- (1) Statistical inference: in this part, it mainly sets up the mathematical model based on data, solves the problem by appropriate method of Queuing theory, and achieves the rationalization of queuing system.
- (2) The inertia of system: namely the probability of regularity of quantity index about queuing, mainly concludes: the distribution of the waiting time of a customer, busying period distribution, the distribution of the queuing length that the customer waiting and so on. It mainly includes two states: the steady state under statistical equilibrium; instantaneous state.
- (3) The system optimization problems: its purpose is to make all systems produce best results, design correctly, and move effectively. In general, the system optimization problem is divided into two categories: the system design optimization and the system control optimization. The former is called a static optimization problem, which goal is to make the system achieve maximum benefit, or under a certain index, the system is the most economical. The latter is called dynamic optimization problem. It is to say, for a given system, how to run to make an objective function value to the optimal.

### III. MODELING

Queuing theory can be divided into single channel queuing system and multi-channel queuing system. This paper mainly researches the performance index under the steady state.

#### A. Single Channel Queuing System

The single channel queuing system is called M/M/1 system. Assume that customers arrive randomly, follows Poisson distribution,  $\lambda$  is the average arrival rate,  $\mu$  is the average output rate,  $\rho = \frac{\lambda}{\mu}$  is traffic intensity or utilization coefficient. When  $\rho < 1$ , the arrival rate is less than the rate of output, and then the intersection traffic will be smooth. If  $\rho \geq 1$ , the arrival rate is greater than the rate of output and then the queuing length will be infinity, the system is not steady. Therefore,  $\rho < 1$  is the necessary and sufficient condition for the system to be steady. Combined with the Little formula, the quantity indexes of single channel queuing system can be obtained, as follows:

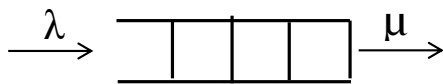


Figure 2. M/M/1 System

- (1) The probability of no vehicle in the system:  $P_0 = 1 - \rho$
- (2) The probability of n vehicles in the system:  $P_n = \rho^n (1 - \rho)$
- (3) The average number of vehicles in the system:  $L = \frac{\rho}{1 - \rho}$
- (4) The average queuing length of vehicles in the system:  $L_q = L \cdot \rho$
- (5) The average staying time of vehicles in the system:  $w = \frac{1}{\mu(1 - \rho)}$
- (6) The average waiting time of vehicles in the system:  $w_q = \frac{L}{\mu}$

#### B. Multi-Channel Queuing System

The multi-channel queuing system is called M/M/N system. Its traffic intensity  $\frac{\rho}{N}$  is which is different from the single channel queuing system. The system is stable when  $\frac{\rho}{N} < 1$ , otherwise, it is not. At the same time,  $\frac{\rho}{N} < 1$  is the necessary and sufficient condition for the system to be steady. In multi-channel Queuing System, use  $\rho = \frac{\lambda}{\mu N}$  instead of  $\rho = \frac{\lambda}{\mu}$  in single channel queuing system.

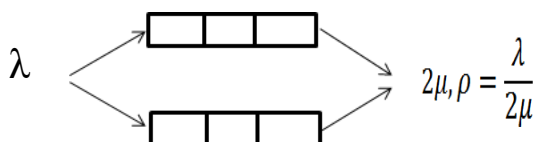


Figure 3. M/M/2 System

- (1) The probability of no vehicles in the system:

$$P_0 = \left[ 1 + \frac{(N\rho)^N}{N!(1-\rho)} + \sum_{n=1}^{N-1} \frac{(N\rho)^n}{N!} \right]^{-1}$$

- (2) The probability of k vehicles in the system:

$$P_0 = \begin{cases} P_0 \frac{(N\rho)^n}{n!}, & n < N \\ P_0 \frac{\rho^n N^N}{N!}, & n \geq N \end{cases}$$

- (3) The probability of queuing:

$$\ell = P(\geq N \text{ vehicles}) = \frac{(N\rho)^N}{N!(1-\rho)} P_0$$

- (4) The average queuing length of vehicles in the system:

$$L_q = \rho \ell (1 - \rho)$$

- (5) The average number of vehicles in the system:

$$L = N\rho + \frac{\rho \ell}{(1 - \rho)}$$

- (6) The average staying time of vehicles in the system:

$$W = \frac{1}{\mu} \left( 1 + \frac{\ell}{N(1-\rho)} \right)$$

- (7) The average waiting time of vehicles in the system:

$$W_q = \frac{L_q}{\lambda}$$

### IV. APPLICATION EXAMPLE

It takes money and people to cut or add the fixed lanes that the number of the lanes should be confirmed in the beginning of construction design. Whether the existing establishment of the lanes is reasonable, the model can validate it. We select the 3 lanes of Pyay road intersection in one direction, take the vehicle flow of July 1, 2019 to July 3, 2019 as research objects. Suppose the time of every vehicle through the intersection is 5s in view of pedestrian and traffic singles. Measure the vehicle flow of July 1, 2014 from 9:00 to 10:00 am is 857, the vehicle flow of July 2, 2014 from 9:00 to 10:00 am is 744, the vehicle flow of July 3, 2014 from 9:00 to 10:00 am is 795. Because of undergoing 45 cycles in one hour, now note the data of the vehicles flow of 3 cycles as shown in the Table I.

TABLE I. TIME MEASURED DATA

Time	1	2	3	4	5	6	7	8
1 Veh	75	54	67	52	54	63	47	60
2 Veh	51	50	43	46	47	51	55	45
3 Veh	45	52	54	48	53	67	58	60
Time	9	10	11	12	13	14	15	Sum
1 Veh	62	59	46	56	58	52	52	857
2 Veh	57	45	55	56	44	55	44	744
3 Veh	49	55	60	45	60	50	39	795

Takes the average traffic volume as standard, we use the model to validate the existing establishment of driveways is reasonable or not.

(1) If the driveway is single, then

$$\lambda = \frac{792}{3600} = \frac{11}{50}, \mu = \frac{1}{5},$$

$$\rho = \frac{\lambda}{\mu} = \frac{11}{10} > 1$$

(2) Conversely, the system metrics of intersection as shown in Table II, the probability of k vehicles in the system is shown in Table III:

TABLE II. SYSTEM METRICS OF INTERSECTION

System	Metrics			
	$L_q$	$L$	$W$	$W_q$
M/M/2	0.477	1.577	7.167	2.168
M/M/3	0.067	1.166	5.303	0.303
M/M/4	0.011	1.111	5.048	0.048
M/M/5	0.002	1.102	5.008	0.009

TABLE III. THE PROBABILITY OF k VEHICLES IN THE SYSTEM

System	$P_k$			
	$P_0$	$P_1$	$P_2$	$P_3$
MM/2	0.290	0.319	0.175	0.096
MM/3	0.327	0.360	0.198	0.073
MM/4	0.334	0.367	0.202	0.074
MM/5	0.333	0.366	0.201	0.074
System	$P_k$			
	$P_4$	$P_5$	$P_6$	$P(k>6)$
MM/2	0.053	0.029	0.016	0.009
MM/3	0.027	0.010	0.004	0.001
MM/4	0.022	0.006	0.002	0.000
MM/5	0.020	0.001	0.000	0.000

By analyzing the data in the Table II, the quantity indexes of system are decreased in the increasing of the driveways. The service indicators of system are in decline with the increasing of the lanes. Therefore the increasing of the lanes has a positive impact on the vehicle flow. By analyzing the data in the Table III, the probability of six or more vehicles is 0 in the intersection in the system of M/M/4. So the 4 driveways is the first selection of the design. 5 driveways are better than 4 driveways. At the same time, the establishment of the driveway needs to consider various factors. The more lanes, the shorter the vehicles queue length. But it will be unnecessary waste in some degree if the scale of the construction is large, the large investment, and the high operating costs. So the system of M/M/3 is the ideal selection to ensure the smooth and fast traffic and save resources. This application example only proves it is feasible that the model can be used to confirm the number of the lanes by analyzing the vehicle flow of the intersection.

### V. CONCLUSION

The paper sets up the queuing model, analyses the traffic flow of intersection through analyzing the queuing theory deeply, and uses the model to analyze the settings of the lane that based on the certain degree of accuracy.

From the paper, the theoretical data is consistent with the reality. Therefore, it is economic that the method of the system metrics in confirming the number of the lanes of the intersection and it can provide references for similar design.

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